1979 Best Paper - Illustrations of Mixing in Laminar Flow

Illustrations of Mixing in Laminar Flow

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I. Introduction

In previous papers, theoretical derivation of a new concept on mixing in laminar flow as function of deformation has been advanced. In the past, it was believed that mixing proceeded linearly with shear. This is only true for a limited number of geometries and is a very inefficient mode of mixing. The theoretical derivation showed that exponenetial rates of mixing are possible. The experimental results in this paper demonstrate this new concept in simple shear flow and elongational flow.

II. SIMPLE SHEAR

A. Experiments

A mixer which can impart simple shear through Couette flow was constructed. A simple schematic diagram is shown in Fig. I. The apparatus imparts an approximation of simple shear by the relative motion of two concentric cylinders (rings a and b in Fig. 1). A shear free boundary condition is maintained on the backing plate by supporting the material being deformed on a layer of liquid metal. The amount of shearing strain imparting an the fluids is linearly proportional to the turning angle. Shear strain can be approximated as

$$S = \frac{L}{w} \qquad (1)$$

where L is the arc length and w is the width between the fixed outer ring and the movable ring.

$$L = R\theta \qquad (2)$$

where R = radius of the movable ring; Theta = turning angle in radian. Therefore,

$$S = \frac{R\theta}{w} \qquad (3)$$

Black and white low density polyethylene were prepared by mixing appropriate colorants with a LDPE resin. The hot colored polymer melt was formed into rings in the annulus of the apparatus. These rings were cut into blocks. Interfaces were created by putting the blocks of LDPE of alternating color into the mixer (see Fig. 2) with the interfaces lying radial in the annulus. The whole apparatus was heated to 350 ° F. The apparatus was taken out of the oven after the polymer melted and thermally equilibrated. An amount of shear strain *g* was imparted on the polymer melt by turning the movable ring. This mixed plastic ring of black and white LOPE was cut into rectangular blocks after it solidified. Measurements were made on these blocks and an average striation thickness was calculated. This value is shown in Fig. 3.

In order to reorientate the interfaces for subsequent shear, the blocks of plastic were put back into the mixer oriented so that the interfaces were again lined up perpendicular to the shearing ring. The melting, shearing, cooling and measuring striation thickness procedures were the same as before. Results of this experiment for differing shears are shown in Fig. 4. Similarly a set of data points were taken with two and three reorientations by repeating the above procedures. These are shown in Figs. 5 and 6. Many sets of data were obtained in a similar fashion with different shears.

B. THEORY

The first fundamental consideration of such mixing of highly viscous liquids was carried out by Spencer and Wiley [4]. They quantitatively related interfacial area growth in plane strain flow with fluid motion and applied this relation to very simple mixing geometries. The growth of an interface in a fluid undergoing shear was found to follow the formula

$$\frac{A_f}{A_o} = \sqrt{1 - 2s \cos\alpha \cos\beta + s^2 \cos^2\alpha} \qquad (4)$$

where s is the magnitude of the shear strain, A_0, the magnitude of the initial area, and alpha and beta are the angles defining the initial orientation of the surface to the shear strain.

Assuming very large unidirectional shear, this leads to a simpler expression

$$\frac{A_f}{A_o} = s \cos\alpha \qquad (5)$$

If the interface is arranged such that it is initially perpendicular to the shearing plane, then

$$\alpha = 0 \qquad (6)$$

$$\cos\alpha = 1$$

Equation (5) becomes

$$\frac{A_f}{A_o} = s \qquad (7)$$

Now, consider a simple shear mixer which subjects material to a large unidirectional shear (s/2), and then distorts the fluid so as to change the orientation of the fluid interface back to the original orientation and subjects it again to shear (s/2). Assuming no effect on the interfacial area during the reorientation process, the total interfacial area of the output at this reorientation is A'. The equation describing the two shearing sections are

$$\frac{A'}{A_o} = \frac{s}{2}\ , \quad \frac{A_f}{A'} = \frac{s}{2} \qquad (8)$$

where A_0 and A_f are the initial and final interfacial area of the mixture respectively after the cycle when a total shear of a is imparted. Combining the equations in (8), we get

$$\frac{A_f}{A_o} = \left(\frac{s}{2}\right)^2 \qquad (9)$$

In general, a simple shear mixer incorporating (N-1) mixing sections, each of which optimize the interface for subsequent shear, and having N shearing sections each of equal shearing magnitude (s/N), causes mixing defined by:

$$\frac{A_f}{A_o} = \left(\frac{s}{N}\right)^N \qquad (10)$$

The relation between the total interfacial area and the average striation thickness of a mixture of constant volume has been derived and given as

$$\text{Volume} = \frac{1}{2} Ar \qquad (11)$$

where r is the striation thickness.

When there is mixing, their states can be related as

$$\text{Volume} = \frac{1}{2} A_o r_o = \frac{1}{2} A_f r_f \qquad (12)$$

Equation (12) leads to the conclusion that

$$\frac{A_f}{A_o} = \frac{r_o}{r_f} \qquad (13)$$

Combining Eqs. (10) and (13) gives

$$\frac{r_o}{r_f} = \left(\frac{s}{N}\right)^N \qquad (14)$$

Defining *g* as the amount of shear strain imparted on the material in each shearing section, we have

$$g = \frac{s}{N} \qquad (15)$$

it results

$$\frac{r_o}{r_f} = g^N \qquad (16)$$

Take logarithm of Eq. (16), we have

$$\log \frac{r_o}{r_f} = N \log g \qquad (17)$$

This equation is shown in Figs. 3, 4, 5 and 6. The slope of the straight lines where N = 1 is 1 and 2 when N = 2 and so on. It shows that the rate of mixing can be improved tremendously by interrupting shearing actions. The interfacial area grows much faster than linearly with one or more reorientations incorporated in the mixer.

The theoretical results on mixing in a simple shear mixers having mixing sections and experimental results show good agreement.

C.DISCUSSION

The results of these very simple experiments generally agree with the theoretical results very well. There are, nevertheless, points which are as high as 10% off the theoretical value. This may be due to the fact that circular planes are used as an approximation of the parallel plane which results variation of shear strain imparted on the fluids or the creation of interfaces when blocks of plastic with lines are put back into the mixer for subsequent shear.

The experiment has demonstrated the effect of reorientation on the rate of mixing. The experiment data provides support on the theoretical analysis of the relation between shear strain, orientation of interfaces, and the interfacial area growth.

III. ELONGATIONAL FLOW

The other kind of deformation of fluid studied in this experiment is elongational flow. The idea that elongational flow gives rapid mixing has not been widely recognized and utilized in mixer design. It is the objective of this experiment to illustrate how mixing is caused by deformation in elongational flow.

A. EXPERIMENT

Pure elongation can be achieved by stretching material as it is done on a tensile test. Silly putty (silicone fluid) was selected for these experiments. Uniform layers of silly putty of different colors were put together Like a sandwich. It was then stretched in the direction of the interfacial plane for an elongation amount λ. The stretched layers of silly putty were folded and then subjected again to the same amount of elongation. This was repeated for seven times and samples were taken at each stage for measurements. Experimental results are plotted in Fig. 7.

B. THEORY

Theory on mixing in elongational flow was presented by Erwin [3]. This equation relates the interfacial area growth to the principal elongation ratios and the initial orientation of the interface by

$$\frac{A}{A_o} = [(\lambda_x \lambda_y)^2 + \cos^2 \alpha \lambda^2 (\lambda_z^2 - \lambda_x^2) + \cos^2 \beta \lambda_x^2 (\lambda_z^2 - \lambda_y^2)]^{1/2} \qquad (18)$$

where (λ_x ,λ_y , λ_z are the principle elongation ratios and α, β define the initial orientation of the interface.

In unaxial elongation, the principle elongation ratios can be expressed as:

$$\lambda_x = \lambda_o$$

$$\lambda_z = \lambda_y = \frac{1}{\lambda_o^{1/2}}$$

Substituting into Eq. (18) yields

$$\frac{A}{A_o} = \left\{\lambda_o + \cos^2 \alpha \left(\frac{1}{\lambda_o^2} - \lambda_o\right)\right\}^{1/2} \qquad (19)$$

If the interface is aligned with the maximum principle value (cos α = 0), Eq. (19) becomes

$$\frac{A}{A_o} = \lambda_o^{1/2} \qquad (20)$$

This has been plotted in Fig. 7. Reasonable agreement with experiment is shown. Since strain is the logarithm of the elongation ratio, linear dependence of mixing on the elongation ration shows exponentional dependence of mixing on strain.

C. DISCUSSION

In order to stretch material uniformly, a uniform thickness is required. Any section with slightly smaller in thickness under elongation will result in non uniform stretching. In this experiment, every effort was made to avoid non uniform stretching. This, however, is the main source of error.

Better results can be obtained if a device which can stretch material uniformly can be constructed. Nevertheless, these rough measurements are sufficient in showing the effect on the rate of mixing in elongation.

IV. CONCLUSION

Mixing in laminar flow is demonstrated in these experiments on simple shear and elongational flows. Measurements demonstrated (1) the linear-relationship of interfacial area growth with shear in non-interrupted flow; (2) higher order of interfacial area growth with shear in interrupted flow; and (3) the exponential interfacial used growth in elongation.

Future work will address the application of this analysis to flows found in real mixers which are much more complex.

V. REFERENCES

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3. L. Erwin, Polym. Eng. Sci., 18, 1048 (1978).
4. R.S. Spencer and R.M. Wiley, J. Colloid Sci., 6, 133 (1951).

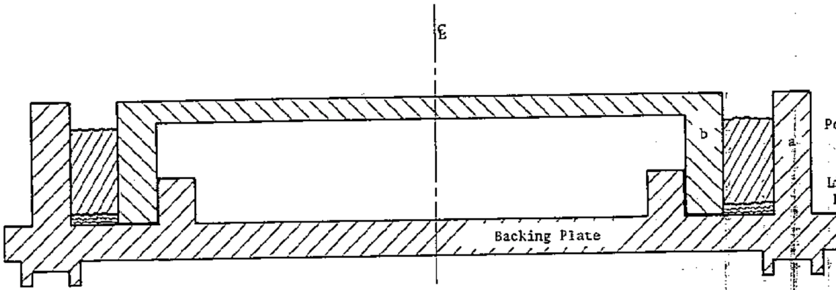

FIGURE 1. Section of Shearing Apparatus

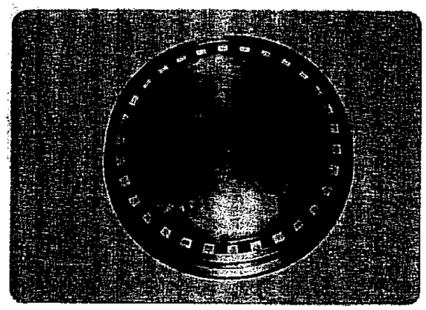

FIGURE 2. Apparatus with material.

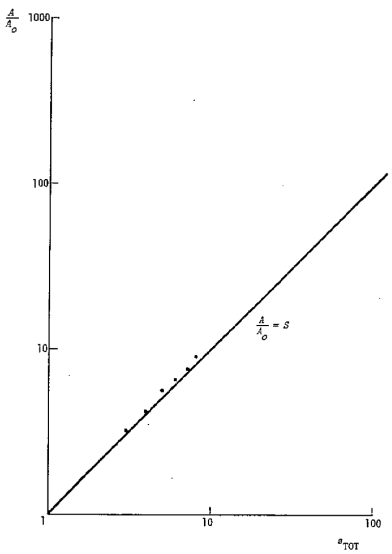

FIGURE 3 Increase in interfacial area as a function of simple shear.

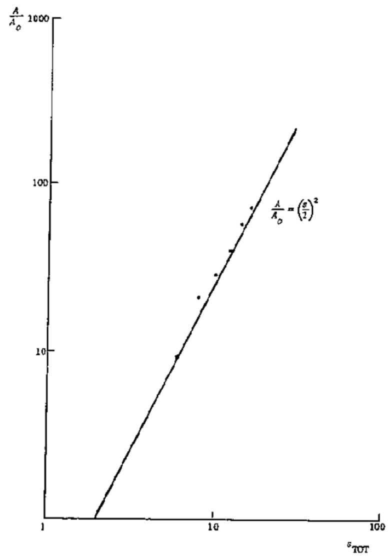

FIGURE 4 Increase in interfacial area as a function of simple shear with one reorientation.

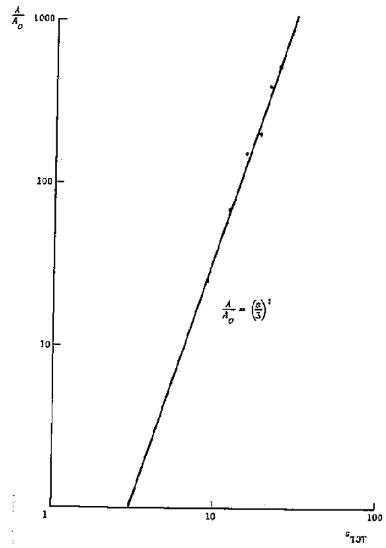

FIGURE 5 Increase in interfacial area as a function of simple shear with two reorientations.

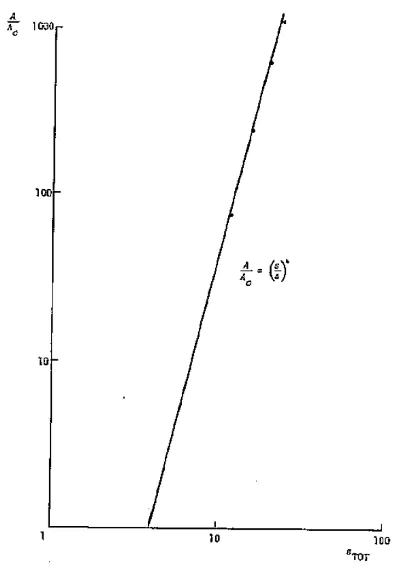

FIGURE 6 Increase in interfacial area as a function of simple shear with three reorientations.

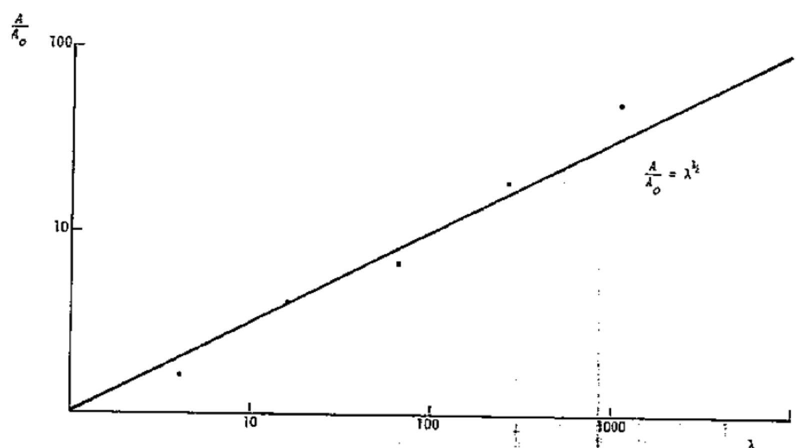

FIGURE 7. Mixing as a function of elongation.

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